Modelling and analysis the volatility of Dow Jones Islamic Indices Returns Using ARCH Models

نمذجة وتحليل تقلبات عوائد مؤشرات داو جونز الإسلامية باستخدام نماذجARCH

Ibtihal BOUHAFS*,

University of Ghardaia, bouhafs.ibtihal@univ-ghardaia.dz,

Received: 2/1/2022 **Accepted**: 5/18/2022 **Published**: 6/5/2022

Abstract:

The purpose of this study is modeling and analysis the volatility of Dow Jones Islamic indices though an application of both symmetric and asymmetric Generalized Autoregressive Conditional Heteroscedastic models and daily data of the Dow Jones Islamic Market index returns during the study period. The results show that Dow Jones Islamic Market index returns have the same commonly observed stylized facts of financial time series. Moreover, the best model for volatility modeling is the PGARCH model.

Keywords: Shari'ah, Filters, Leverage Effect, Models, GARCH.

JEL Classification Codes: C13; C58; G10; G41.

الملخص:

تهدف الدراسة إلى نمذجة وتحليل تقلبات مؤشر داو جونز الإسلامي، مستخدمة في ذلك نماذج الانحدار الذاتي المعمم المشروط بعد تجانس التباين المتناظرة وغير المتناظرة، وذلك بالإعتماد على البيانات اليومية لعوائد مؤشر داو جونز الإسلامي خلال فترة الدراسة الممتدة من 2010/01/04 إلى 2020/15/05. أشارت النتائج أن عوائد مؤشر داو جونز الإسلامي لها نفس الحقائق النمطية الموجودة في السلاسل الزمنية المالية، كما أظهرت النتائج أن أفضل نموذج لنمذجة التقلبات هو نموذج.PGARCH

الكلمات المفتاحية: شريعة؛ تصفية؛ أثر رافعة؛ نماذج؛.GARCH

تصنيف جال: C13; C58; G10; G41.

**Corresponding author.*

1. INTRODUCTION

Islamic finance provides investment opportunities in the financial markets respecting the Islamic principles (Chiadmi & Ghaiti , 2014)through inventing products and financial tools that serve individuals and organizations in getting a financial portfolio that goes with their religious principles (EL Khamlichi, 2012). In 1999, many international financial markets launched Islamic equity indexes that reflect the prices movement of traded Islamic products after filtering them from shares that oppose the Islamic principles(Besseba & Benchiha, 2019).

Recently, many studies have focused on the series of the Islamic equity indexes and their characteristics. (Chiadmi & Ghaiti , 2014) indicate that the Islamic equity indexes have been significantly impacted by the financial crisis, but to a lesser degree than the traditional ones. (Rejeb & Arfaoui, 2019) indicate that the risks degree in the Islamic equity indexes is bigger than the traditional ones. In another study, (Chiadmi, 2015) found that Islamic equity indexes have almost all the statistical characteristics noticed in financial markets. These characteristics are known in many studies as stylized facts.

Due to the existence of these characteristics in time series, (Bollerslev, 1986) found a generalized model from ARCH models named GARCH. However, the latter is based only on the characteristic of similar effect of shocks. In the same context, many practical studies showed the need for other models that take into consideration the dissimilar effects of the changing variance resulting from shocks. Other studies showed that asymmetric GARCH models are the best for modeling the indexes fluctuations such as the study of (Sahnoune & Benlaib, 2019) that showed that asymmetric GARCH models outperform the symmetric ones. Moreover, the study of (Ben Nasr & Ajmi, 2014) found that FITVGARCH models work better than FIGARCH models in estimating and modeling the conditional fluctuations of the index returns. Thus, the problematic of the paper can be stated as follows:

What is the best model among GARCH models for modeling the volatility of DJIM index returns during the period of 2010 -2020?

Before arriving to answer this problematic, we hypothesize that:

1. Daily returns of DJIM index are characterized with negative skewness and high kurtosis during the study period.
2. DJIM index returns are not independent from each other during this period.
3. GARCH models are suitable to estimate the fluctuations of the DJIM index during the period of the study.
4. Negative shocks have a bigger impact on fluctuations of the DJIM index returns compared to the positive shocks.

- **Aims of the study:**

This study aims at:

- Trying to suggest a standard model for modeling volatility of the DJIM index returns using GARCH models and all what may help in making the necessary decisions.

- Trying to know the statistical characteristics of the Islamic equity indexes.
- Measuring the relation between the return and risk in the DJIM index during 2010-2020.

➢**Importance of the study:**

The study draws its importance from the fact that it helps investors, operators, and financial managers in managing estimations of the Islamic equity indexes and setting trading strategies based on fluctuations and, thus, better manage risks. Moreover, the importance of the study lies within the fact that it aims at defining the behavior of the returns of DJIM index that are important for Muslim and non-Muslim investors.

2. Theoretical background of the Islamic equity indexes and methodology of building them:

Islamic equity indexes have drawn the interest of many Muslim and non-Muslim investors with conservative beliefs because these indexes suit their ethical principles. Muslim Muftis allowed Muslims to invest in the financial assets that meet specific principles that aim at reducing the incompatible activities. This led the fund managers to set selection criteria whose characteristics do not convene with Islamic principles (Majidi, 2016). Consequently, Islamic equity indexes are built on the standard criteria after the following series of filters:

2.1 Qualitative filtering: This criterion filters the institutions qualitatively on the level of their activities (Majidi, 2016, p. 145). The Islamic rules prohibit investment in the sectors whose products represent a danger for the human health and whose consumption is not allowed in Islam. This encompasses all what is called “illegal activities” (EL Khamlichi, 2012, p. 88). Islamic rules prohibit investment in sectors such as weapons, alcohol, tobacco, drugs, pigs’ meat, pornography, gambling, and investments in financial, conventional, and insurance institutions that give loans or borrow. Moreover, the official committee publishes the list of the sectors that do not go with the investment principles in Islamic finance (EL Khamlichi & Viallefont, 2015, p. 6).

2.2Quantitative filtering: At this stage, we apply the standards of the financial ratios allowed in the institutions indebtedness that had been chosen at the first phase. It measures 3 types of ratios concerning the financial structure of the institution (Chiadmi, 2015, p. 70). These financial ratios differ from one index to another and are not agreed upon by the official committees. They are no more than extreme limits that are allowed (EL Khamlichi, 2012, p. 91) because they are mentioned in Quran or Sunna (Majidi, 2016, p. 145).

2.2.1 Debts ratio: This ratio is the total debt / total assets or average value of market capitalization during the year. It allows excluding all the institutions with high indebtedness and is one of the basic principles that characterize the Islamic finance from the traditional through prohibiting the interest-rate transactions. Therefore, the debts levels are taken into consideration as there is a consensus that 33.33% is the maximum level of debts (Majidi, 2016, p. 146).

It is noteworthy indicating that all ratios use the total debt in the numerator. However, the denominator expresses two orientations: indexes using the market value (DJIM and S&P) and indexes using the total assets (MSCI and FTSE) The official committee differentiates between the total market value and the total assets and takes the one whose value is higher as a denominator for the index (EL Khamlichi, 2012, p. 93).

2.2.2Filtering receivables: It is calculated through the residual of dividing total debtors on the market value of the last 12 months. This ratio must be less than 33% from the market value and, thus, if the size of the debtor in every institution is more than 33% from the market value, it will not be accepted in the portfolio that goes with Islam. On the other hand, if a big part of the institution's assets is debtor, the trade assets of the institutions are dominated by cash flows, with the risk of not collecting receivables (Chiadmi, 2015, p. 70).

2.2.3 Filtering the liquidity generating profit: This ratio is calculated by dividing liquidity plus the securities generating profits by the market value of the last 12 months(Chiadmi, 2015, p. 70). It is based on a basic principle of prohibiting interests from Islamic finance. As a result, finance alternatives have been established far from the conventional ones. When institutions deposit the surplus of liquidity in conventional banks in the countries that do not have Islamic banks, the official committees intervene to determine the maximum limit that the institutions must maintain. This filter is different on the side of the allowed limit as a liquidity that can be deposited in banks; it is from 33% for Islamic indexes such as Dow Jones, P&P, and Stoxx to 70% for MSCI index. This difference explains that liquidity deposit is allowed as long as it respects the limits that do not generate revenues in the form of interests (EL Khamlichi, 2012, p. 93).

3. Method and tools:

3.1 Study variables: Data of the practical study are made up of daily time series of DJIM index returns -closing prices- for the period of 04/01/2010 to 15/05/2020. This period has been chosen due to the frequency of financial crises and uncertainty from one side, and the availability of data during the study period from another side. Moreover, this index is one of the main effective indexes in the Islamic financial markets. Data of the series were from: quotes.wsj.com.

3.2 Theoretical background of the used models:

3.2.1 Symmetric GARCH models: Symmetric models consider that the conditional variance depends on the size of the shock and not on its sign (Namugaya, Weke, & Charles, 2014, p. 5175). Among these models, we find:

- GARCH model : Bollerslev (1986) suggested GARCH model to reduce the number of the power coefficients, from the infinite number of coefficients, into a small number. Thus, he could exclude ARCH model (Soualili & Belghait , 2018, p. 3). Among the simplest characteristics of this model is GARCH (1, 1) (Namugaya, Weke, & Charles, 2014, p. 5175);

$$\boldsymbol{r_t} = \mu + \varepsilon_t$$
$$\boldsymbol{\sigma_t^2} = \omega + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2$$

r_t is defined asthe real returns in Time t and μ is the average expected returns. The conditional variance must be positive.

-GARCH-M model: Engle and Robins (1987) suggested GARCH-M model so that the conditional variance be a variable that explains the conditional mean. Thus, it becomes ready to describe the effect of the speed of the fluctuations on the return of the financial assets. The formulation of the model is written as such (Namugaya, Weke, & Charles, 2014, p. 5175):

$$\boldsymbol{r_t} = \mu + C\sigma_t^2 + \varepsilon_t$$

$C$is the coefficient of risk premiums. If it is positive, it indicates that the returns are correlated to their fluctuations in a positive way i.e. the increase in the return mean is the outcome of the increase in the conditional variance as an alternative for the increasing risks.

- FIGARTH model: Baillie, Bollerslev, and Mikkelsen (1996) suggested FIGARTH that models only the case where the decrease of the correlation coefficients takes the form of hyperbola and is written as such (Chikhi, Bebdob, & Bendob, 2017, p. 261):

$$\boldsymbol{\vartheta(L)} = 1 - \frac{1}{\beta(L)}(1-L)^d \quad , \qquad 0 \leq d \leq 1$$

However, it isthe only that is characterized with a fast decrease in the lapse factor, and this is what we can call the long memory.

3.2.2 Asymmetric GARCH models: They appeared because of the criticism faced by symmetric GARCH models because they were based on the symmetric effect of the shock. Among these models, we find:

- EGARCH model:It was presented by Nelson (1991). The function of the conditional variance is exponential and nonlinear contrary to what Bollerslev sees in GARCH model (Chikhi, Bebdob, & Bendob, 2017, p. 258).The fluctuation increases after the negative shocks more than after the positive shocks at the same level; it is called the leverage effect. It ensures that conditional variance is always positive even if the values of the parameters are negative (Namugaya, Weke, & Charles, 2014, p. 5176). This model is characterized withnot requiring constraints toguarantee the non-negative conditional variance through giving a formality in the exponential form. The equation of the model is written as such (Nelson, 1991, p. 352):

$$\boldsymbol{\sigma_t^2} = \exp\left(\omega + \gamma z_{t-1} + \alpha(|z_{t-1}| - E|z_{t-1}|) + \beta \ln(\sigma_{t-1}^2)\right) \cdots\cdots\cdots (6)$$

-TGARCH model: Jaganathan & Runkle (1993) and Glosten & Zakoian (1994) suggested TGARCH model to express the leverage effect in the quadratic form contrary to EGARCH model that is expressed in its exponential form (Matei, 2009, p. 53) The equation of the model is as such (Namugaya, Weke, & Charles, 2014, p. 5176):

$$\boldsymbol{\sigma_t^2} = \omega + \alpha_1\beta_{t-1}^2 + \gamma d_{t-1}\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2 \cdots\cdots\cdots (7)$$

Where $d_{t-1}$is a dummy variable, i.e.

$$d_{t-1} = \begin{cases} 1, & if\,\varepsilon_{t-1} < 0, \quad badnews \\ 0, & if\,\varepsilon_{t-1} \geq 0, \quad goodnews \end{cases}$$

Coefficient γ represents the effect of the financial leverage. The model decreases to GARCH model when $\gamma = 0$. Contrarily, when the shock is positive (good news), the effect on the fluctuation isα_1. But, if the shock is negative (bad news), the effect on the fluctuation is$\alpha_1 + \gamma$. Moreover, when γ is big and positive, the negative shock would have a bigger effect on σ_t^2 than the positive shock.

-GJR-GARCH: It was developed by Glosten, Jagannathan, and Runkle (1993). They studied the relation between the expected value and the fluctuations of the excessive nominal returns. It was noted that the effect of the positive shocks is different than the effect of the negative shocks. Thus, they provided a suggestion of adding a dummy variable to the variance equation to test the positive and negative effect of the shock (Al-Ahmad & Kusai Salman, 2019, p. 533). The following equation illustrates that (Sahnoune & Benlaib, 2019, p. 533):

$$\boldsymbol{\sigma_t^2} = \omega + (\alpha + \gamma I_{t-1})\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2 \cdots\cdots\cdots (8)$$

$$I_{t-1} = \begin{cases} 1 \; if \, \varepsilon_{t-i} < 0 \\ 0 \;\; if \, \varepsilon_{t-i} \geq 0 \end{cases}$$

- The Power GARCH model: It was developed by Schwert (1989) and Taylor (1986). The conditional standard deviation was used as a measure for the fluctuations instead of the conditional symmetry. After that, it was generalized by Dind, Granger, and Engel (1993) to focus on the characteristic of asymmetry through adding the power coefficient δ in the modeling. Thus, the equation is written as such (Wiphatthanananthakul & Sriboonchitta, 2010, p. 144):

$$\boldsymbol{\sigma_t^\delta} = \omega + \sum_{i=1}^{p} \alpha_i(|\mu_{t-1}| - \gamma_i\mu_{t-1})^\delta + \sum_{j=1}^{q} \beta_j\sigma_{t-j}^\delta \cdots\cdots\cdots (9)$$

$$\delta > 0, |\gamma_i| \leq 1 \, for i = 1,2, \cdots r$$

$$\gamma_i = 0 \, for i > r, \quad r \leq p$$

If $\gamma \neq 0$, the model captures the dissimilar effects. PGARCH decreases to GARCH when $\delta = 2$ and $\gamma_i = 0$.

4. RESULTS AND DISCUSSION:

4.1 Study of the statistical characteristics of the return series of the DJIM index:

A descriptive study has been carried out on the DJIM index returns using the descriptive statistics by central tendency and dispersion measures. But, before studying the statistical characteristics and modeling DJIM index, the return of the index has been calculated as such:

$$R_t = \ln\left(\frac{S_t}{S_{t-1}}\right)$$

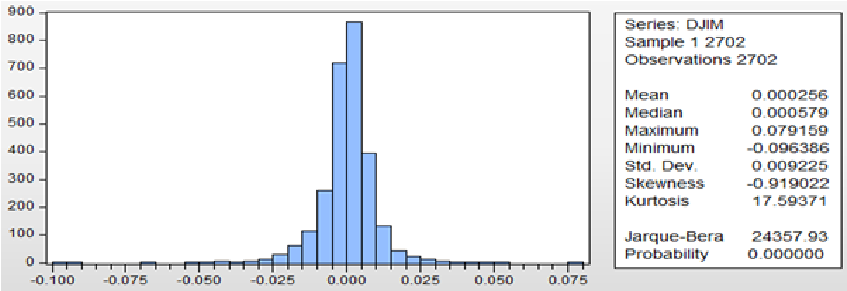
Where:

R_t: Index return in time t,

S_t: Index value in t,

S_{t-1}: Index value in time t-1.

Table 1. Statistical analyses of DJIM returns during study period

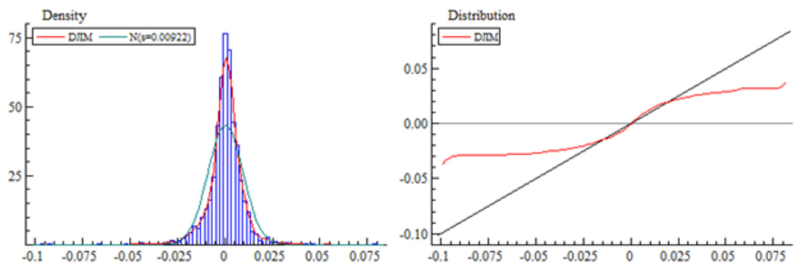


Source: EViews 10 output.

Results of the statistical returns of DJIM index in table (01) show that the index achieved a return mean of 0.0002. Furthermore, the table shows a low value of the standard deviation, but, higher than return mean. This indicates that investment in this index is subject to big risks.

Results of Skewness coefficient show that the returns are characterized with a negative skewness and are centered to the left. This indicates theexistence of a big probability that the returns are negative. Besides, the returns during the study period are characterized with high kurtosis which justifies the problem of fat tails as kurtosis coefficient exceeded the value of the three that face the normal distribution. This means the deviation of thereturns series from the normal distribution through concentration of the distribution more around the mean. This is confirmed by the results of Jarque-Bera test that indicate that returns of the DJIM index did not follow the normal distribution during the study period. The following figure illustrates that:

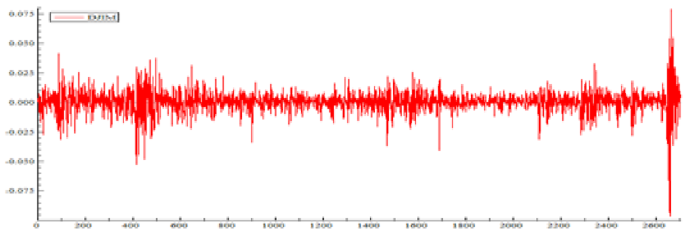
Fig1. Normal distribution of the daily return series of DJIM index



Source: OxMetrics 6 output.

We see from figure (2) the existence of remarkable fluctuation ofDJIM index returns and a concentration of the sharp volatilities and an increase in the number of peaks either negatively or positively that reflect the effect of the numerous shocks on the index.

Fig2. Movement of the daily returns of DJIM during 2010-2020



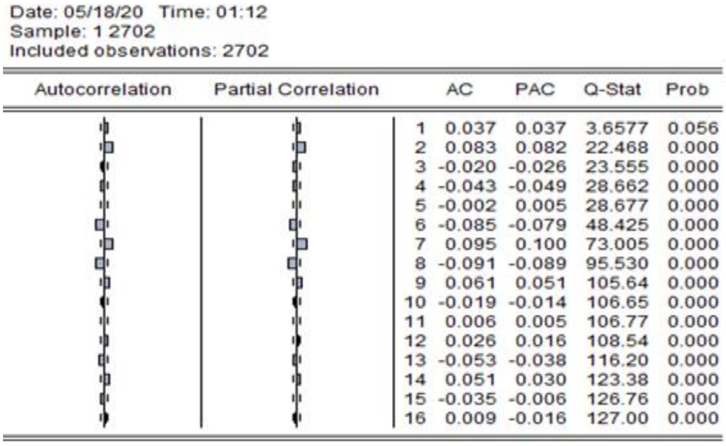
Source: OxMetrics 6 output.

4.2 Analysis of the autocorrelation and tests of unityrooton the returns series of DJIM index:

4.2.1 Significance of autocorrelation test coefficients:

Figure (3) shows that Q (k) statistic calculated for the last value in column Q-Statof DJIM index returns is bigger than the scheduler statisticofchi-squared distribution at a freedom degree of 16 at the level of significance 5%. Thus, we refuse the null hypothesis$H_0$and accept the alternative hypothesisH_1about the non-null autocorrelation coefficients, andhence, the index returns are not independent from each other.

Fig3. testing the significance of autocorrelation test coefficients


Date: 05/18/20 Time: 01:12
Sample: 1 2702
Included observations: 2702

	AC	PAC	Q-Stat	Prob
1	0.037	0.037	3.6577	0.056
2	0.083	0.082	22.468	0.000
3	-0.020	-0.026	23.555	0.000
4	-0.043	-0.049	28.662	0.000
5	-0.002	0.005	28.677	0.000
6	-0.085	-0.079	48.425	0.000
7	0.095	0.100	73.005	0.000
8	-0.091	-0.089	95.530	0.000
9	0.061	0.051	105.64	0.000
10	-0.019	-0.014	106.65	0.000
11	0.006	0.005	106.77	0.000
12	0.026	0.016	108.54	0.000
13	-0.053	-0.038	116.20	0.000
14	0.051	0.030	123.38	0.000
15	-0.035	-0.006	126.76	0.000
16	0.009	-0.016	127.00	0.000

Source:EViews 10 output.

4.2.2Static and stabilitytests: Results ofADF and PP tests in table (2) show the absence of unit rootin daily data series of DJIM index returns. Results show that all the calculated values were lower than the tabulated value at a significance level of 5%; thus, confirming the stabilityof the series.

Table 2. Results of stability test of DJIM index returns

ADF			
Theory t	**Without Intercept and trend**	**With Intercept and without trend**	**With Intercept and trend**
At level 5%	-1.9409	-2.8624	-3.4114
T calculated	-19.4862	-19.5430	-19.5401
PP			
Theory t	**Without Intercept and trend**	**With Intercept and without trend**	**With Intercept and trend**
At level 5%	-1.9409	-2.8624	-3.4114
T calculated	-50.0402	-50.0540	-50.0449

Source:EViews 10 output.

4.3Estimation of the fluctuations of the index returns using symmetric and asymmetric GARCH models:

After the independence and stability study had shown that DJIM index can predict its future returns relying on the series of the previous returns, we can continue predicting through GARCH model.

4.3.1 Estimation of ARMA model and ARCH effect: The following table shows the results of applying ARMA model on the returns of DJIM index.

Table 3. Estimation of ARMA (1.1) model

Dependent Variable: DJIM
Method: ARMA Maximum Likelihood (OPG - BHHH)
Date: 05/18/20 Time: 01:30
Sample: 1 2702
Included observations: 2702
Convergence achieved after 65 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.000256	0.000218	1.175171	0.2400
AR(1)	0.461808	0.109300	4.225152	0.0000
MA(1)	-0.411387	0.112064	-3.670997	0.0002
SIGMASQ	8.48E-05	8.90E-07	95.31641	0.0000

R-squared	0.003224	Mean dependent var	0.000256
Adjusted R-squared	0.002115	S.D. dependent var	0.009225
S.E. of regression	0.009216	Akaike info criterion	-6.534367
Sum squared resid	0.229132	Schwarz criterion	-6.525630
Log likelihood	8831.930	Hannan-Quinn criter.	-6.531207
F-statistic	2.908595	Durbin-Watson stat	2.033535
Prob(F-statistic)	0.033356		

Source:EViews 10 output.

We notice from results of table (3) that the model is statisticallyaccepted due to the significance of thelinear regression and the moving average. The value of prob is less than 0.05 and, thus, we refuse the null hypothesis H_0that says that the parameters are not significant, and accept the alternative hypothesis H_1that says that the parameters of the model have a statistical significance at the significance levelof 5%.

Using the results above, we test the condition of inconstancy of errors variance in the studied series. We relied onLM-ARCH test. Results are shown in the following table:

Table 4. Results of ARCH effect test on the daily returns of DJIM index

Heteroskedasticity Test: ARCH

F-statistic	401.1402	Prob. F(1,2699)	0.0000
Obs*R-squared	349.4938	Prob. Chi-Square(1)	0.0000

Test Equation:
Dependent Variable: RESID^2
Method: Least Squares
Date: 05/18/20 Time: 01:32
Sample (adjusted): 2 2702
Included observations: 2701 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	5.43E-05	6.43E-06	8.453331	0.0000
RESID^2(-1)	0.359712	0.017960	20.02848	0.0000

R-squared	0.129394	Mean dependent var	8.48E-05
Adjusted R-squared	0.129072	S.D. dependent var	0.000348
S.E. of regression	0.000324	Akaike info criterion	-13.22817
Sum squared resid	0.000284	Schwarz criterion	-13.22380
Log likelihood	17866.65	Hannan-Quinn criter.	-13.22659
F-statistic	401.1402	Durbin-Watson stat	2.304745
Prob(F-statistic)	0.000000		

Source:EViews 10 output.

From table (4), it seems that there is an ARCH-effect in the series of the residuals at significance level 1% during the study period. Thus, the nullhypothesis $H_0$is refused and the alternative hypothesis H_1, that sates that there is an ARCH effect, is accepted. Therefore, the variance in the returnseries is not constant through time and we can apply GARCH model to solve this problem.

4.3.2 Estimation of GARCH model: From table (5), we see results of GARCH model (1,1) estimation of DJIM index returns during the study period under the hypothesis of student distribution of errors that is considered statistically accepted at significance level of 5%.

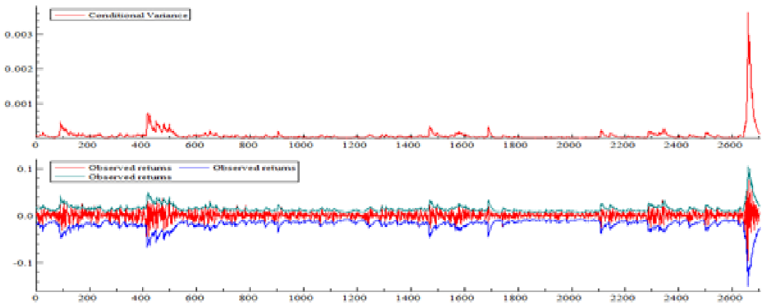
Table 5. Estimation of GARCH model (1,1)

```
 ********************************
 ** G@RCH( 1) SPECIFICATIONS **
 ********************************
Dependent variable : DJIM
Mean Equation : ARMA (1, 0) model.
No regressor in the conditional mean
Variance Equation : GARCH (1, 1) model.
No regressor in the conditional variance
Student distribution, with 5.12967 degrees of freedom.

Strong convergence using numerical derivatives
Log-likelihood = 9564.92
Please wait : Computing the Std Errors ...

 Robust Standard Errors (Sandwich formula)
                      Coefficient  Std.Error  t-value  t-prob
Cst(M)                   0.000754 0.00012004    6.282  0.0000
AR(1)                    0.107910   0.018840    5.728  0.0000
Cst(V) x 10^6            1.494759    0.38045    3.929  0.0001
ARCH(Alpha1)             0.148507   0.021213    7.001  0.0000
GARCH(Beta1)             0.843534   0.018884    44.67  0.0000
Student(DF)              5.129671    0.51007    10.06  0.0000

No. Observations :       2702  No. Parameters   :        6
Mean (Y)         :    0.00026  Variance (Y)     :  0.00009
Skewness (Y)     :   -0.91902  Kurtosis (Y)     : 17.59371
Log Likelihood   :   9564.918  Alpha[1]+Beta[1]:  0.99204
```

Source: OxMetrics6 output.

From the table, we see that GARCH model (1,1) is statistically accepted at significance level of1%. The significance value of the coefficient a_1 (ARCH effect) indicates the existence of shock effects on the conditional fluctuations of DJIM indexreturns, i.e. the fluctuations are too sensitive to any incident in the financial market.GARCH effect indicates that the variance resulting from the high value of the returns will be followed by a high variance in the later period. Total of ARCH and GARCH coefficients is almost 1 and this indicates the continuity of fluctuations shocks.This value confirms the cluster characteristic of the variance as the high variance will be followed by another high variance in a later period. Thus, the shock goes to infinity. Figure (4) illustrates the clear increase in the conditional variance of the returns of the index under study in the last period that was characterized with high fluctuations due to COVID-19 repercussions on the world economy.

Fig.3. Conditional Variance of DJIM index returns



Source: OxMetrics6 output.

4.3.3 GARCH-M model estimation:

This model is used to measure the relationbetween the return and the risk. It includes the equationof the conditional variance in the mean equation.

Table 6. Estimation of GARCH-M model (1.1)

```
 ******************************
 ** G@RCH( 2) SPECIFICATIONS **
 ******************************
Dependent variable : DJIM
Mean Equation : ARMA (1, 0) model.
No regressor in the conditional mean
Variance Equation : GARCH (1, 1) model.
            in-mean
No regressor in the conditional variance
Student distribution, with 5.10994 degrees of freedom.

Strong convergence using numerical derivatives
Log-likelihood = 9566.62
Please wait : Computing the Std Errors ...

 Robust Standard Errors (Sandwich formula)
                  Coefficient  Std.Error  t-value  t-prob
Cst(M)               0.000555 0.00016332    3.398  0.0007
AR(1)                0.106647   0.018886    5.647  0.0000
Cst(V) x 10^6        1.492623    0.37402    3.991  0.0001
ARCH(Alpha1)         0.147863   0.020815    7.104  0.0000
GARCH(Beta1)         0.844101   0.018338    46.03  0.0000
Student(DF)          5.109944    0.50571    10.10  0.0000
ARCH-in-mean(var)    4.909909     2.6915    1.824  0.0682

No. Observations  :       2702  No. Parameters    :        7
Mean (Y)          :    0.00026  Variance (Y)      :  0.00009
Skewness (Y)      :   -0.91902  Kurtosis (Y)      : 17.59371
Log Likelihood    :   9566.623  Alpha[1]+Beta[1]: :  0.99196
```

Source: OxMetrics 6 output.

The results of the return equation through GARCH-M of the index under study show the existence of a high statistical significance of the parameters. The meanequation shows the existence of a positive sign for the parameter ofARCH which indicates that there is a direct relationship between the return and the risk when investing in the index of shares prices of DJIM.

4.3.4Estimation of TGARCH, PGARCH, GJR-GARCH, and EGARCH models: These models show whether the good and bad news have the same effect on the fluctuations, and thus, measure the financial leverage effect.

- Estimation of EGARCH model: table (7) shows EGARCH model (1.1) which shows the characteristic of dissimilarity of effects of the shocks (leverage effect) on DJIM index returns.

Table 7. Estimation of EGARCH model (1.1)

```
 ******************************
 ** G@RCH( 1) SPECIFICATIONS **
 ******************************
Dependent variable : DJIM
Mean Equation : ARMA (1, 0) model.
No regressor in the conditional mean
Variance Equation : EGARCH (1, 1) model.
No regressor in the conditional variance
Student distribution, with 4.79722 degrees of freedom.

Weak convergence (no improvement in line search) using numerical derivatives
Log-likelihood = 9579.5
Please wait : Computing the Std Errors ...

 Robust Standard Errors (Sandwich formula)
                  Coefficient  Std.Error  t-value  t-prob
Cst(M)               0.000442 0.00011660    3.790  0.0002
AR(1)                0.110061   0.017735    6.206  0.0000
Cst(V) x 10^6        0.039548    0.38800   0.1019  0.9188
ARCH(Alpha1)         0.173379    0.17941   0.9664  0.3339
GARCH(Beta1)         0.936842  0.0082993    112.9  0.0000
EGARCH(Theta1)      -0.203070   0.036751   -5.526  0.0000
EGARCH(Theta2)       0.174591   0.026817    6.510  0.0000
Student(DF)          4.797218    0.41364    11.60  0.0000

No. Observations :       2702  No. Parameters   :        8
Mean (Y)         :    0.00026  Variance (Y)     :  0.00009
Skewness (Y)     :   -0.91902  Kurtosis (Y)     : 17.59371
Log Likelihood   :   9579.500
```

Source: OxMetrics6 output.

From results of the table, we find that EGARCH models are statistically accepted and show that powers are accepted and significant at levels 1% and 5 %, and the parameter γ(leverage coefficient) got a negative value what makes us infer the existence of the leverage effect, i.e. fluctuations of DJIM index returns increase after the negative shocks -bad news- more than after the positive shocks -good news- from the same level.

- Estimation of GJR-GARCH model: Estimation results shown in table (8) indicate that GJR- model is statistically accepted and that the coefficient of leverage effectγ is positive. Thus, this result confirms EGARCH models, i.e. negative shocks have a higher effect on the conditional variance than the positive shocks of the same size.

Table 8. Estimation of GJR-GARCH model (1.1)

```
 ********************************
 ** G@RCH( 3) SPECIFICATIONS **
 ********************************
Dependent variable : DJIM
Mean Equation : ARMA (1, 0) model.
No regressor in the conditional mean
Variance Equation : GJR (1, 1) model.
No regressor in the conditional variance
Student distribution, with 5.52498 degrees of freedom.

Strong convergence using numerical derivatives
Log-likelihood = 9616.14
Please wait : Computing the Std Errors ...

 Robust Standard Errors (Sandwich formula)
                  Coefficient  Std.Error  t-value  t-prob
Cst(M)               0.000521 0.00011904    4.380  0.0000
AR(1)                0.105750   0.018416    5.742  0.0000
Cst(V) x 10^6        1.835887    0.36434    5.039  0.0000
ARCH(Alpha1)        -0.016768   0.012998   -1.290  0.1971
GARCH(Beta1)         0.856614   0.016992    50.41  0.0000
GJR(Gamma1)          0.268649   0.039536    6.795  0.0000
Student(DF)          5.524977    0.59489    9.287  0.0000

No. Observations :       2702  No. Parameters  :         7
Mean (Y)         :    0.00026  Variance (Y)    :   0.00009
Skewness (Y)     :   -0.91902  Kurtosis (Y)    :  17.59371
Log Likelihood   :   9616.143
```

Source: OxMetrics6 output.

- Estimation of PGARCH model: This model provides a confirmation of the existence of the dissimilar shocks in DJIM index returns through the usage of the standard deviation for modeling rather than the variance to estimate δpower. The following table shows the results:

Table 9. Estimation of PGARCH model (1.1)

Dependent Variable: DJIM
Method: ML ARCH - Student's t distribution (BFGS / Marquardt steps)
Date: 05/18/20 Time: 01:34
Sample (adjusted): 2 2702
Included observations: 2701 after adjustments
Failure to improve likelihood (non-zero gradients) after 122 iterations
Coefficient covariance computed using outer product of gradients
Presample variance: backcast (parameter = 0.7)
@SQRT(GARCH)^C(7) = C(3) + C(4)*(ABS(RESID(-1)) - C(5)*RESID(-1))^C(7) + C(6)*@SQRT(GARCH(-1))^C(7)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.000449	0.000118	3.790255	0.0002
AR(1)	0.108421	0.019220	5.640990	0.0000
Variance Equation				
C(3)	0.000564	0.000301	1.872265	0.0612
C(4)	0.111220	0.008171	13.61164	0.0000
C(5)	0.999928	2.1E-104	4.8E+103	0.0000
C(6)	0.891437	0.009793	91.02672	0.0000
C(7)	0.838993	0.104575	8.022857	0.0000
T-DIST. DOF	6.008090	0.723967	8.298842	0.0000

R-squared	-0.004129	Mean dependent var	0.000255
Adjusted R-squared	-0.004501	S.D. dependent var	0.009227
S.E. of regression	0.009248	Akaike info criterion	-7.125541
Sum squared resid	0.230817	Schwarz criterion	-7.108062
Log likelihood	9631.043	Hannan-Quinn criter.	-7.119220
Durbin-Watson stat	2.158993		

Source: EViews 10 output.

- Estimation of TGARCH model: It is different than GJR-GARCH model in its modeling of the conditional standard deviation instead of the conditional variance. Table (10) shows its outcomes on DJIM index returns.

Table 10. Estimation of TGARCH model (1.1)

Dependent Variable: DJIM
Method: ML ARCH - Student's t distribution (BFGS / Marquardt steps)
Date: 05/18/20 Time: 01:39
Sample (adjusted): 2 2702
Included observations: 2701 after adjustments
Convergence achieved after 38 iterations
Coefficient covariance computed using outer product of gradients
Presample variance: backcast (parameter = 0.7)
GARCH = C(3) + C(4)*RESID(-1)^2 + C(5)*RESID(-1)^2*(RESID(-1)<0) + C(6)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.000522	0.000122	4.274034	0.0000
AR(1)	0.105945	0.019684	5.382217	0.0000
Variance Equation				
C	1.86E-06	2.89E-07	6.450918	0.0000
RESID(-1)^2	-0.016193	0.012747	-1.270398	0.2039
RESID(-1)^2*(RESID(-1)<0)	0.269708	0.030065	8.970982	0.0000
GARCH(-1)	0.855128	0.014314	59.74178	0.0000
T-DIST. DOF	5.570750	0.655527	8.498128	0.0000

R-squared	-0.004100	Mean dependent var	0.000255
Adjusted R-squared	-0.004472	S.D. dependent var	0.009227
S.E. of regression	0.009248	Akaike info criterion	-7.111761
Sum squared resid	0.230811	Schwarz criterion	-7.096466
Log likelihood	9611.433	Hannan-Quinn criter.	-7.106230
Durbin-Watson stat	2.153057		

Source:EViews 10 output.

From the table, we see that TGARCH model is statistically accepted as the powers are accepted and guaranteed at level of 5%. Furthermore, we see that the negative shock has a big effect on $\sigma_t^2$of the positive shock because the value of the coefficient $\gamma$is big and positive.

-Estimation of FIGARCH model: One of themodels used in testing the long memory in DJIM index returns is FIGARCH (1, d, 1). Its estimation results on the DJIM index returns indicate that it is statistically accepted and that the parameters are significant and that the fractional calculusparameter d ranges between values 0 and 10. This proves the existence of a long memory in the fluctuations of DJIM indexreturns with a continuity of shocks in it.

Table 11. Estimation of FIGARCH model (1.d.2)

```
 ******************************
 ** G@RCH( 1) SPECIFICATIONS **
 ******************************
Dependent variable : DJIM
Mean Equation : ARMA (1, 0) model.
No regressor in the conditional mean
Variance Equation : FIGARCH (1, d, 2) model estimated with BBM's method (Truncation order
No regressor in the conditional variance
Student distribution, with 5.54953 degrees of freedom.

Strong convergence using numerical derivatives
Log-likelihood = 9570.42
Please wait : Computing the Std Errors ...

 Robust Standard Errors (Sandwich formula)
                    Coefficient  Std.Error  t-value  t-prob
Cst(M)                 0.000755 0.00012051    6.264  0.0000
AR(1)                  0.109562   0.019011    5.763  0.0000
Cst(V) x 10^6          0.674779    0.42811    1.576  0.1151
d-Figarch              0.248146   0.075637    3.281  0.0010
ARCH(Phi1)             0.730965   0.061654    11.86  0.0000
ARCH(Phi2)             0.212340   0.047996    4.424  0.0000
GARCH(Beta1)           0.898893   0.040763    22.05  0.0000
Student(DF)            5.549533    0.55215    10.05  0.0000

No. Observations :       2702  No. Parameters  :        8
Mean (Y)         :    0.00026  Variance (Y)    :  0.00009
Skewness (Y)     :   -0.91902  Kurtosis (Y)    : 17.59371
Log Likelihood   :   9570.416
```

Source: OxMetrics6 output.

4.3.5 Choosing the best model for the estimation of the index returns: In this phase, we shall choose the best model for modeling the fluctuation of DJIMindexreturns series. To do so, we relied on information standards AIC, SIC, HQC. Table (12) shows the values of the estimated models according to the random error distribution of the models.

Table 12. Choosing the best model for the estimation of DJIM indexreturns fluctuations

	GARCH	GARCH-M	EGARCH	GJR-GARCH	PGARCH	TGARCH
AIC	-7.0754	-7.0759	-7.0847	-7.1126	-7.1255	-7.1117
SIC	-7.0623	-7.0606	-7.0672	-7.0973	-7.1080	-7.0964
HQC	-7.0707	-7.0704	-7.0784	-7.1070	-7.1192	-7.1062

Source: EViews 10 output.

From table (12), we see that the estimation of PGARCH (1,1) model according to the distribution T-student's was thebest due to the lowest values for AIC, SIC, HQC.

5. CONCLUSION

This study tried to find the best model for modeling the fluctuations of DJIMindex returns relying on the symmetric and asymmetric GARCH models.The study used the daily data of closing prices of DJIMindex returns during the period 2010-2020. Findings show that:

- ✓ There is an increase in the mean of DJIMindex returns followed by highfluctuations which reflect the degree of risks in this index.
- ✓ DJIMindex returns are characterized with kurtosis and skewness towards the left, and thus, returns do not follow the normal distributionduring the study period. This is a problem related to the behavior of the investors; **thus, we accept the first hypothesis.**
- ✓ Series ofDJIM index returns are characterized with independence and absence of unity root which indicates that returns are not random; **thus, we refuse the second hypothesis.**
- ✓ Series of DJIM index returns were characterized with changing variance which consolidates the use of the conditional asymmetric models.
- ✓ Symmetric GARCH models are statistically accepted and could cope with DJIMindex returns fluctuations. We found that the fluctuations are very sensitive to any financial market incident and that any strong shock in the fluctuations currently would have a long effect on the expected future values of the fluctuation.
- ✓ GARCH models helped in the analysis of the characteristic of clusterfluctuation in the time series of DJIMindex, i.e.these models could analyze the characteristic of fluctuation; thus, **we accept the 3rd hypothesis.**
- ✓ GARCH-M model is statistically accepted at significance level 5%. This result indicates the existence of a direct relation between the returns and the risks when investing in the Islamic shares. The higher the risk level is for the investor, the higher the return level asked in return ofthis investment is.
- ✓ Asymmetric GARCH models proved that DJIMindex returns are characterized withincreasing conditional variance. When there is a negative shock, the fluctuation size is big compared to the level of change in the fluctuation after the positive shocks. The investor takes rapid decisions thatcan impact the supply and demand in the market to avoid any other potential losses and risks. Moreover, the investor may not take decisions that have that effect after positive shocks; thus, these models could analyze the characteristic of leverage effect. Therefore, **we accept the 4th hypothesis.**
- ✓ PGARCH model (1,1) is the best among the estimated models in measuring DJIM index returns fluctuations. This, implicitly, means that these models have the ability to include the different effects resulting from the sudden negative shocks from the urgent political and economic news.
- ✓ There is a statistical significance of a long memory in the fluctuations of DJIM index returns. This indicates the strong effect of shocks on the market fluctuations.

Based on what has been said, we suggest:

- ✓ Taking into consideration the statistical characteristics of the Islamic equity indexes through using the models that have mostof these characteristics.
- ✓ Paying more attention to the predictive econometric study of the Islamicequity indexes.

✓ The possibility of studying the dynamic conditional correlation between the fluctuations of the Islamicequity indexes and the conventional.

6. Bibliography List:

1. **Books:**

-Chikhi, M., & Bendob, A. (2017),**Financial Econometrics and its applications in financial markets**, DAR Al Hamed For publication and distribution, Algeria.

2. **Theses:**

- Chiadmi, M. (2015). **La volatilité des indices boursiers islamiques dans le contexte de la crise financière** (Doctoral dissertation). Ecole Mohammadia d'Ingénieurs.

- EL Khamlichi, A. (2012). **Éthique et performance: le cas des indices boursiers et des fonds d'investissement en finance islamique** (Doctoral dissertation). Ferrand I: Université d'Auvergne-Clermont

- Majidi, E. (2016). **Finance islamique et croissance économique: quelles interactions dans les pays MENA** (Doctoral dissertation). Université de Pau.

3. **Journal article:**

- Al- Ahmad, Z., & Kusai Salman, A. (2019),**Modeling Volatility in Emerging Stock Markets: The Case of Damascus Securities Exchange**, Tishreen University Journal for Research and Scientific Studies -Economic and Legal Sciences Series, 41(2).

- Ben Nasr, A., & Ajmi, A. (2014),**Modelling the volatility of the Dow Jones Islamic Market World Index using a fractionally integrated time-varying GARCH (FITVGARCH) model**, Applied Financial Economics, 24(14).

- Besseba, A., & Benchiha, K. (2019),**Analysis the Effect of Exchange Rate Volatility on Islamic Stock Indices Performance: An Empirical Study of the Dow Jones Islamic Market**, Dirassat Journal Economic Issue, 11(1).

- Bollerslev, T. (1986), **Generalized autoregressive conditional heteroskedasticity**, Journal of econometrics,31(3).

- Chiadmi, M., & Ghaiti , F. (2014),**Modeling volatility of Islamic stock indexes: Empirical evidence and comparative analysis**, DLSU Business & Economics Review, 24(1).

- EL Khamlichi, A., & Viallefont, A. (2015), **La performance des indices boursiers en finance islamique: Une Meta-analyse,** Revue de Gestion et d'Économie, 3(1).

- Matei, M. (2009),**Assessing volatility forecasting models: why GARCH models take the lead**, Romanian Journal of Economic Forecasting, 12(4).

- Namugaya, J., Weke, P., & Charles, W. (2014),**modelling stock returns volatility on Uganda securities exchange**, Applied Mathematical Sciences, 8(104).

- Nelson, D. (1991),**Conditional heteroskedasticity in asset returns: A new approach**, Econometrica: Journal of the Econometric Society, pp. 347-370.

- Rejeb, A., & Arfaoui, M. (2019),**Do Islamic stock indexes outperform conventional stock indexes? A state space modeling approach**, European Journal of Management and Business Economics, 28(3).

- Sahnoune, S., & Benlaib, B. (2019),**Volatility Modeling of Islamic Stock Indices Returns Using GARCH Models**, El-Bahith Review, 19(1).

- Soualili , S., & Belghait , B. (2018),**Modeling the daily return volatility of the CAC 40 index by applying the APGARCH model**, Revue des Réformes Economiques et Intégration En Economie Mondiale, 12(26).
- Wiphatthananthakul, C., & Sriboonchitta, S. (2010),**The Comparison among ARMA-GARCH,-EGARCH,-GJR, and-PGARCH models on Thailand Volatility Index**, The Thailand Econometrics Society, 2(2).