

PI-Augmented Super-Twisting Observer for Air Leak Fault Reconstruction in PEMFC Supply Manifolds

Asma Rahmani¹, Mohamed Bougrine¹, Atallah Benalia¹

¹LACoSERE Laboratory, University of Amar Telidji, Laghouat 03000, Algeria

Corresponding author: asma.rahmani@lagh-univ.dz
<https://doi.org/10.34118/ajeee.v1i1.4886>

Abstract

This paper focuses on proton exchange membrane fuel cells (PEMFCs) and their air supply system. The study investigates the application of advanced observers, specifically higher-order sliding mode observers with PI injection, to precisely estimate the partial pressure of oxygen in PEMFC air supply systems. These observers continuously monitor real-time system behavior and detect abnormal conditions, including faults such as sudden air manifold leaks. The primary objective of this research is to enhance the reliability, durability, and efficiency of PEMFCs by detecting faults in their air supply system, which is crucial for achieving optimal performance.

Keywords: Proton Exchange Membrane Fuel Cell (PEMFC), Sliding Mode Observer, PI injection, Fault Diagnosis and Isolation (FDI).

1. Introduction

Fuel cells generate electricity through the reaction of hydrogen and oxygen. In Proton Exchange Membrane Fuel Cells (PEMFCs), the anode and cathode sides are respectively supplied with hydrogen and oxygen. The hydrogen in fuel cell vehicles is stored in pressurized tanks while air serves as the source of oxygen, pumped into the cathode by a compressor. The air supply system presents a significant challenge to the overall performance of PEMFC systems [1, 2]. Functioning similarly to an autonomous power plant in vehicles, the compressor motor is powered by the PEMFC itself. Hence, the net power of the system is the difference between the power produced by the fuel cell and that consumed by the air supply system, with negligible consumption from other auxiliary systems [3, 4]. Experimental studies have indicated that the air supply system can consume up to 30% of the fuel cell's power under high load conditions. Therefore, it must operate optimally, delivering just enough oxygen required for the hydrogen and oxygen reaction [5]. Figure 1 illustrates a block diagram of a standard PEMFC system commonly used in automotive applications.

Unfortunately, achieving such control is not feasible without precise knowledge of the partial pressure of oxygen [6]. Conventional sensors can only provide the total air pressure inside the cathode, which includes partial pressures of other gas mixtures such as nitrogen, carbon dioxide, and others. Inaccurate knowledge of oxygen quantity in the cathode can lead to severe issues, such as oxygen depletion during rapid

load transitions. Observers can serve two critical roles in the air supply system. Firstly, they can provide an accurate estimation of the partial pressure of oxygen [7]. Secondly, leveraging available sensors and mathematical knowledge of the model, they can detect and identify abnormal behavior, thereby detecting faults and failures effectively [8]. Among the potential faults, leaks in the air manifold represent a significant issue. These leaks can adversely affect the optimal performance of the PEMFC by disrupting the distribution of air essential for the electrochemical reaction. Such issues may be associated with the deterioration of the sealing mechanisms in the cathode inlet nozzle valve or with the occurrence of fractures in the associated piping [9, 10].

Several research studies have been conducted to develop methodologies for fault detection, identification, and analysis in PEM fuel cell systems [11, 12]. Laghrouche et al. presented an observer-based fault reconstruction method for PEM fuel cells, extending the results to nonlinear uncertain systems with Lipschitz nonlinearities [8]. Liu et al. developed a robust model-based fault diagnosis approach for PEM fuel cell air-feed systems, specifically addressing sudden air leaks in the air supply manifold [13]. Zhang et al. introduced a data-driven fault diagnosis approach for PEMFC systems in hybrid trams based on deep learning [14]. Yang et al. presented a robust fault diagnosis and fault-tolerant control strategy for PEMFC systems using an augmented LPV observer [15].

This work specifically focuses on the detection and diagnosis of leaks in the air manifold of Proton Exchange Membrane Fuel Cells (PEMFC) using advanced observers. Liu's work on model-based fault diagnosis for the PEM fuel cell air-feed system has laid a solid foundation in this field. A robust diagnostic approach based on a modified sliding mode algorithm is examined for a class of uncertain nonlinear systems. The modified super-twisting sliding mode algorithm proposed by Liu [13], which comprises two nonlinear terms and two linear terms, is employed to estimate the system states and simultaneously reconstruct the fault signal, with a particular emphasis on detecting leaks in the air manifold. We build upon Liu's theoretical framework by introducing an application scenario involving a sudden air leak in the supply manifold where the fault scenario is simulated by introducing a new airflow rate. This algorithm enhances diagnostic precision. Subsequently, the fault signal is reconstructed from the equivalent output error injection term, computed online from the system model outputs and the observer, enabling rapid and accurate fault identification.

This paper is structured as follows: Section 2 details the

modelization of the PEMFC air feed subsystem. Section 3 introduces the modified super-twisting algorithm for fault detection and isolation. In Section 4, the application of this method to the PEMFC air feed system, including the integration of a sudden air leak fault, is discussed. Simulation results are presented to illustrate the practicality and efficiency of the proposed method. Section 6 concludes the paper.

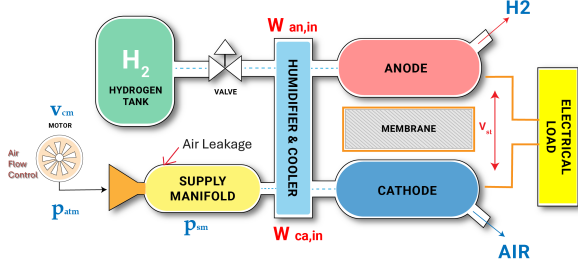


Figure 1: PEM fuel cell system scheme.

2. PEMFC Air Feed Subsystem Modelization

The modeling of the fuel cell system, with particular emphasis on the air supply subsystem, plays a pivotal role in understanding, optimizing, and controlling the performance of these innovative energy systems. The model proposed in this article aims to simplify the system's complexity while retaining its key influential elements. The simplified model presented consists of only four states, capturing essential interactions within the fuel cell air supply system [16]. These states encompass the dynamics of the compressor, pressure variation in the air manifold, and phenomena within the cathode. Such simplification enables a clearer understanding and more efficient analysis of system behavior. However, for maximum accuracy, more comprehensive models incorporating up to nine states are available in the literature [17]. This lays the groundwork for observer synthesis for monitoring and diagnostics.

The objective of the air supply system is to regulate the quantity of oxygen in the cathode. It typically consists of an electromechanical air compressor that maintains the required oxygen pressure and mass flow rate into the cathode of the PEMFC [18]. This system can consume up to 30% of the fuel cell stack power and demands precise control to optimize the net power output of the PEMFC system. The system has been modeled under the assumptions of ideal gas behavior, uniform temperature across the cathode and coolant, controlled inlet reactant humidity, availability of high-pressure hydrogen, vapor-phase water within the cathode, and negligible gas diffusion layer flooding. Spatial variations are disregarded, treating flow channels and gas diffusion layers as a unified volume, with controlled anode pressure to match the cathode pressure [16].

2.1. Air supply manifold model

The air supply manifold functions as a short-term air reservoir, ensuring a controllable air supply to the fuel cell. The

mass flow rates at the inlet and outlet are determined by the compressor flow rate W_{cp} and the supply manifold flow rate $W_{sm,out}$, respectively. The air supply manifold model is characterized by

$$\frac{dp_{sm}}{dt} = \frac{R_a T_{cp,out}}{V_{sm}} (W_{cp} - W_{sm,out}) \quad (1)$$

where V_{sm} represents the volume of the supply manifold, and $T_{cp,out}$ denotes the temperature of the air exiting the compressor. If the pressure difference is minimal, the flow rate can be approximated as

$$W_{sm,out} = k_{sm,out}(p_{sm} - p_{ca}) \quad (2)$$

2.2. Compressor model

The air compressor is used to supply oxygen to the cathode side of the fuel cell system [19]. The model's inputs include the intake air pressure $p_{cp,in}$, the intake air temperature $T_{cp,in}$, and the downstream pressure, which is the supply manifold pressure p_{sm} . The intake air is typically atmospheric, with an assumed pressure and temperature of $p_{atm} = 1$ and $T_{atm} = 25^\circ$, respectively. The dynamic state in the model, represented by the compressor speed ω_{cp} , is given by the following equation:

$$\frac{d\omega_{cp}}{dt} = \frac{1}{J_{cp}} (\tau_{cm} - \tau_{cp} - f\omega_{cp}) \quad (3)$$

where J_{cp} is the inertia of the motor and the compressor ($kg.m^2$), τ_{cm} is the torque of the motor ($N.m$), τ_{cp} is the load torque imposed by the compressor ($N.m$) and f is the friction coefficient.

$$\tau_{cm} = \eta_{cm} k_t I_q \quad (4)$$

and

$$\tau_{cp} = \frac{C_p T_{atm}}{\omega_{cp} \eta_{cp}} \left[\left(\frac{p_{sm}}{p_{atm}} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right] W_{cp} \quad (5)$$

where η_{cm} is the mechanical efficiency of the motor, k_t is the motor constant, I_q is the quadrature current of the motor, C_p is the specific heat capacity of air ($1004 J.kg^{-1}.K^{-1}$), γ is the ratio of specific heats of air (equal to 1.4), and W_{cp} is the mass flow rate of the compressor

The mass flow rate W_{cp} of the twin-screw compressor depends on its angular velocity ω_{cp} . The relationship is given by:

$$W_{cp} = \frac{1}{2\pi} \eta_v V_{cpr/tr} \rho_a \omega_{cp} \quad (6)$$

here η_{v-c} is the volumetric efficiency, $V_{cpr/tr}$ is the volume compressed per revolution, and ρ_a is the air density [2]. Table 2 provides the parameters used in the compressor model.

2.3. Oxygen and nitrogen model

Thermodynamic properties and mass conservation principles are utilized to model the behavior of air within the cathode [20]. Considering the mass continuity of oxygen and nitrogen inside the cathodic volume and applying the ideal gas law, the following relationships are established:

$$\dot{p}_{O_2} = \frac{RT_{fc}}{M_{O_2} V_{ca}} (W_{O_2,in} - W_{O_2,out} - W_{O_2,rc}) \quad (7)$$

$$\dot{p}_{N_2} = \frac{RT_{fc}}{M_{N_2} V_{ca}} (W_{N_2,in} - W_{N_2,out}) \quad (8)$$

$W_{O_2,in}$ denotes the mass flow rate of oxygen entering the cathode, while $W_{O_2,out}$ represents the mass flow rate of oxygen exiting it. $W_{O_2,rct}$ indicates the rate at which oxygen is consumed due to reactions. Similarly, $W_{N_2,in}$ and $W_{N_2,out}$ denote the mass flow rates of nitrogen entering and exiting the cathode, respectively. M_{O_2} and M_{N_2} refer to the molar masses of oxygen and nitrogen, and V_{ca} represents the volume of the cathode. These quantities are essential in modeling the thermodynamic properties and mass conservation within the cathodic environment. The cathode inlet flow rate $W_{ca,in}$ is assumed to be equal to the outlet flow rate from the supply manifold $W_{sm,out}$, as specified in equation (2)

$$W_{ca,in} = k_{ca,in}(p_{sm} - p_{ca}) \quad (9)$$

Here, the cathodic pressure p_{ca} is assumed to be spatially invariant, equivalent to the sum of the partial pressures of oxygen, nitrogen, and water vapor.

$$p_{ca} = p_{O_2} + p_{N_2} + p_{sat}(T_{fc}) \quad (10)$$

2.4. The state-space equations

Based on Eqs. (1), (3), (7), and (8), the dynamics of the cathodic part of the fuel cell are governed by the following four states [16, 21–24]:

$$\dot{x}_1 = c_1(x_3 - p_{ca}) - \frac{c_3 x_1 W_{ca,out}}{c_4 x_1 + c_5 x_2 + c_6} - c_7 I_{st} \quad (11)$$

$$\dot{x}_2 = c_8(x_3 - p_{ca}) - \frac{c_3 x_2 W_{ca,out}}{c_4 x_1 + c_5 x_2 + c_6} \quad (12)$$

$$\dot{x}_3 = c_{14} \left(1 + c_{15} \left[\left(\frac{x_3}{c_{11}} \right)^{c_{12}} - 1 \right] \right) [W_{cp} - c_{16}(x_3 - p_{ca})] \quad (13)$$

$$\dot{x}_4 = -c_9 x_4 - \frac{c_{10}}{x_4} \left[\left(\frac{x_3}{c_{11}} \right)^{c_{12}} - 1 \right] W_{cp} + c_{13} u \quad (14)$$

The stack current I_{st} is conventionally regarded as a disturbance [18], while the control input u represents the motor quadrature current. The measured outputs and the performance variables of the system are given by

$$y = [W_{cp}, T_{cp}, p_{sm}, W_{ca,out}, I_{st}, V_{st}]^T \quad (15)$$

where P_{net} is the net power of the fuel cell stack, and λ_{O_2} is the oxygen excess ratio, defined as the ratio of the oxygen entering the cathode to the oxygen reacting within the fuel cell. It is written as

$$\lambda_{O_2} = \frac{W_{O_2,in}}{W_{O_2,rct}} = \frac{c_{18}(p_{sm} - p_{ca})}{c_{19} I_{st}} \quad (16)$$

For safety and efficiency optimization reasons, it is typical to operate the fuel cell with an oxygen excess ratio λ_{O_2} around 2, regardless of the operating point defined by the current demand [17], [22]. It should be noted that a λ_{O_2} value above 2 results in lower efficiency because the excess oxygen supplied to the cathode leads to wasted energy from the compressor [25]. Conversely, a value below 2 leads to oxygen starvation phenomenon [26]. The definition of system parameters $c_1, \dots, c_8$ based on physical parameters and their respective values are illustrated in Table 1 and Table 2 respectively.

3. Modified Super Twisting for Fault Detection and Isolation

It is helpful to recall the super-twisting sliding mode observer (ST-SMO) algorithm [13]. Consider a nonlinear system that includes both linear and nonlinear parts and is subject to a fault $f(t)$:

$$\dot{z} = Az + G(z, u) + F(y, u)f(t) \quad (17)$$

$$y = Cz \quad (18)$$

where $z = [z_1, z_2]^T \in \mathbb{R}^n$, $z_1 \in \mathbb{R}^p$, $z_2 \in \mathbb{R}^{n-p}$, $u(t) \in U \subset \mathbb{R}^m$ (system input), and $y \in Y \subset \mathbb{R}^p$ (output vector). A is a constant matrix defined as:

$$A = \begin{bmatrix} A_1^{p \times p} & A_2^{p \times (n-p)} \\ A_3^{(n-p) \times p} & A_4^{(n-p) \times (n-p)} \end{bmatrix}, \quad A \in \mathbb{R}^{n \times n} \quad (19)$$

where A_4 is Hurwitz stable, and $C \in \mathbb{R}^{p \times n}$ is a full-rank matrix ($q \leq p < n$). The function $G(z, u)$ is Lipschitz continuous with respect to z , and $D(y, u)$ is a smooth, bounded function. The fault signal $f(t)$ satisfies $\|f(t)\| \leq f_{max}$ and $\dot{f}(t)$ is bounded.

Reordering the state variables, the system becomes:

$$\begin{cases} \dot{z}_1 = A_1 z_1 + A_2 z_2 + G_1(z, u) + F_1(y, u)f(t) \\ \dot{z}_2 = A_3 z_1 + A_4 z_2 + G_2(z, u) \\ y = z_1 \end{cases} \quad (20)$$

$$\text{where } G(z, u) = \begin{bmatrix} G_1(z, u) \\ G_2(z, u) \end{bmatrix} \text{ and } F(y, u) = \begin{bmatrix} F_1(y, u) \\ 0 \end{bmatrix}$$

Assumption 1 The nonlinear functions $G_1(z, u)$ and $G_2(z, u)$ are assumed to be Lipschitz continuous with respect to z_2 ; specifically,

$$\|G_i(z_1, z_2, u) - G_i(z_1, \hat{z}_2, u)\| \leq \gamma_i \|z_2 - \hat{z}_2\| \quad (21)$$

Where $i \in \{1, 2\}$ are predetermined positive constants [13], [27].

An ST SMO (Sliding Mode Observer) for this system is designed as:

$$\begin{cases} \dot{\hat{z}}_1 = A_1 y + A_2 \hat{z}_2 + G_1(\hat{z}, u) + v(y - \hat{y}) \\ \dot{\hat{z}}_2 = A_3 y + A_4 \hat{z}_2 + G_2(\hat{z}, u) \\ \hat{y} = \hat{z}_1 \end{cases} \quad (22)$$

where v is the modified ST term injection [28].

$$v(s) = \underbrace{k_1 |s|^{\frac{1}{2}} \text{sign}(s) + k_2 \int_0^t \text{sign}(s) d\tau}_{\text{Super-Twisting Classic}} + \underbrace{k_3 s + k_4 \int_0^t s d\tau}_{\text{linear PI injection}} \quad (23)$$

where $k_i, i \in \{1, 2, 3, 4\}$, are positive constants to be determined.

Let $e_y = y - \hat{y}$ and $e_2 = z_2 - \hat{z}_2$. By subtracting Eq (22) from Eq (20), the error dynamical system is obtained as follows:

$$\dot{e}_y = -v(e_y) + A_2 e_2 + \Delta G_1 + F_1(y, u)f(t) \quad (24)$$

$$\dot{e}_2 = A_4 e_2 + \Delta G_2 \quad (25)$$

where $\Delta G_1 = G_1(z, u) - G_1(\hat{z}, u)$ and $\Delta G_2 = G_2(z, u) - G_2(\hat{z}, u)$

Proposition 1 under the premise of Assumption 1, the error dynamical system Eq (24) achieves exponential stability provided there exists a positive definite matrix P satisfying the following inequality:

$$A_4^T P + P A_4 + \frac{1}{\varepsilon} P P^T + \varepsilon \gamma_2^2 I_{n-p} + \sigma I_{n-p} \leq 0 \quad (26)$$

ε and σ are two small positive quantities.

Proof We utilize the proof of Theorem 1 from [13], [28]. rigorously demonstrated that the term $A_2 e_2 + \Delta G_1 + F_1(y, u)f(t)$ is bounded.

3.1. Fault reconstruction

According to the theorem 1 e_y and $\dot{e}_y$ converge to zero within a finite time [29]. Consequently, the system enters a sliding motion where, $e_y = \dot{e}_y = 0$, leading to the following relationship:

$$v(e_y) = A_2 e_2 + \Delta G_1 + F_1(y, u)f(t) \quad (27)$$

From Proposition 1, it follows that:

$$\lim_{t \rightarrow \infty} \|A_2 e_2 + \Delta G_1\| \leq (\|A_2\| + \gamma_1) \|e_2(t)\| = 0 \quad (28)$$

Thus, assuming the matrix $F_1(y, u)$ is invertible, the fault signal can be estimated as

$$\hat{f}(t) = F_1^{-1}(y, u)v(e_y) \quad (29)$$

4. Application to the PEMFC Air Feed System

Reconsidering the four-state dynamic model of the PEMFC presented in the modeling section, introducing a fault W_{loss} into the dynamics of the air collector. For the application of the detection algorithm, it is sufficient to consider only the states p_{O_2} , p_{N_2} , and p_{sm} . The dynamics of ω_{cp} are excluded from the model since their impact on the other states is mediated through the function W_{cp} , which is directly measured. Therefore, the faulty model is reformulated as follows:

$$\begin{cases} \dot{x}_1 = c_1(x_3 - p_{ca}) - \frac{c_3 x_1 W_{ca, out}}{c_4 x_1 + c_5 x_2 + c_6} - c_7 I_{st} \\ \dot{x}_2 = c_8(x_3 - p_{ca}) - \frac{c_3 x_2 W_{ca, out}}{c_4 x_1 + c_5 x_2 + c_6} \\ \dot{x}_3 = c_{14} \left(1 + c_{15} \left(\left(\frac{x_3}{c_{11}} \right)^{c_{12}} - 1 \right) \right) \\ \quad \times [W_{cp} - c_{16}(x_3 - p_{ca}) - f(t)] \end{cases} \quad (30)$$

with the state vector

$$x = [p_{O_2} \ p_{N_2} \ p_{sm}]^T \quad (31)$$

and the measured outputs

$$y = [W_{cp}, T_{cp}, p_{sm}, W_{ca, out}, I_{st}]^T \quad (32)$$

In establishing an analogy between Eq (31) and (22), we have:

$$\begin{cases} z_1 = x_3 = p_{sm} \\ A_1 = 0 \\ A_2 = [0 \ 0] \\ F_1 = -c_{14} \left(1 + c_{15} \left(\left(\frac{x_3}{c_{11}} \right)^{c_{12}} - 1 \right) \right) \\ G_1 = -F_1 (W_{cp} - c_{16}(x_3 - p_{ca})) \end{cases} \quad (33)$$

And for the variable z_2

$$\begin{cases} z_2 = [x_1 \ x_2]^T \\ A_3 = [c_1 \ c_8]^T \\ A_4 = \begin{bmatrix} -c_1 & -c_1 \\ -c_8 & -c_8 \end{bmatrix} \\ G_2(z, u) = \begin{bmatrix} -c_1 c_2 - \frac{c_3 x_1 W_{ca, out}}{c_4 x_1 + c_5 x_2 + c_6} - c_7 I_{st} \\ -c_8 c_2 - \frac{c_3 x_2 W_{ca, out}}{c_4 x_1 + c_5 x_2 + c_6} \end{bmatrix} \end{cases} \quad (34)$$

The modified Super-Twisting observer applied to the fuel cell assumes the following form:

$$\begin{cases} \dot{\hat{x}}_1 = c_1(\hat{x}_3 - p_{ca}) - \frac{c_3 \hat{x}_1 W_{ca, out}}{c_4 \hat{x}_1 + c_5 \hat{x}_2 + c_6} - c_7 I_{st} \\ \dot{\hat{x}}_2 = c_8(\hat{x}_3 - p_{ca}) - \frac{c_3 \hat{x}_2 W_{ca, out}}{c_4 \hat{x}_1 + c_5 \hat{x}_2 + c_6} \\ \dot{\hat{x}}_3 = c_{14} \left(1 + c_{15} \left(\left(\frac{\hat{x}_3}{c_{11}} \right)^{c_{12}} - 1 \right) \right) \\ \quad \times [W_{cp} - c_{16}(\hat{x}_3 - p_{ca})] + v(s) \end{cases} \quad (35)$$

$v(s)$ is the injection term (4) such as $s = e_y = p_{sm} - \hat{p}_{sm}$

4.1. Integration of the sudden air leak fault in the PEMFC air feed system

In various studies, the issue of air leakage within the air feed system of PEMFCs has been approached by modifying the constant c_{16} , referred to the physical constant $k_{sm, out}$ [8], [30–33]. Principally, it is inaccurately represents the sudden air leak by pretending an enlargement of the existing orifice that discharges into the cathode. It has a significant limitation.

Inspired by the principles of the nozzle equation, a sudden leak should be represented as a new mass flow rate that explicitly accounts for the pressure difference between the inside of the system and the external environment [9]. However, the fault signal $f(t)$ is simulated by introducing a new flow rate into the pressure dynamics of the air collector, denoted as:

$$f(t) = W_{loss} \quad (36)$$

$$\dot{p}_{sm} = F_1 [W_{cp} - W_{ca, in} - W_{loss}] \quad (37)$$

Where,

$$W_{\text{loss}} = k_{\text{smf}}(p_{\text{sm}} - p_{\text{atm}}) \quad (38)$$

The value of k_{smf} represents the intensity of the fault.

5. Simulation Results

5.1. The faultless observer

The observer is initially implemented and tested using a four-state battery model without any fault scenarios. The physical parameters of the stack utilized are provided in Table 2. It is important to note that in our approach, five variables are assumed to be measured: the injected air flow rate into the collector (W_{cp}), its temperature (T_{cp}), the pressure (p_{sm}), which is the only measured state of the system, the outgoing air flow rate from the cathode ($W_{ca,out}$), and the stack current (I_{st}). Except for the collector pressure, all measured quantities are either disturbances to the system (exogenous inputs) such as I_{st} , or state functions (endogenous inputs) including W_{cp} , T_{cp} , and $W_{ca,out}$. Although the measurement of the latter variables is not strictly necessary for the implementation of our observation and diagnostic approach, given that their forms are known, they are nonetheless very useful in reducing model uncertainties, which have a direct impact on the observer's efficiency. The state variables of the battery model have been initialized as follows:

$$\mathbf{x} = [p_{O_2} \quad p_{N_2} \quad p_{sm} \quad \omega_{cp}]^T = [0.11 \quad 0.85 \quad 1.48 \quad 5100]^T \quad (39)$$

Where the pressures are in bar and the speed is in rad/s. The estimated state variables were initialized at values 50% lower. The system was subjected to a piecewise constant current I_{st} ranging from 100 to 300A (Figure 2) to test the observer's convergence across all operating points of the system. The compressor voltage (control variable) was provided by a PI controller that regulates the oxygen equivalence ratio λ_{O_2} to the reference value $\lambda_{O_2}^{ref} = 2$. The figures depict the evolution of the variables I_{st} , v_{cm} , λ_{O_2} , $\hat{p}_{O_2}$ vs p_{O_2} , $\hat{p}_{N_2}$ vs p_{N_2} , $\hat{p}_{sm}$ vs p_{sm} , and the fault signal f , respectively.

It demonstrates that the controller effectively stabilizes the oxygen excess ratio around its reference value, $\lambda_{O_2}^{ref} = 2$, exhibiting strong disturbance rejection of the current I_{st} . Figures 5, 6, and 7 illustrate the exponential convergence of the observer with precise tracking of the actual states despite their rapid evolution. Despite initially distant conditions, the observer converged in less than one second, a negligible duration for our application.

5.2. The fault scenario

The fault scenario involving a sudden air leak in the supply manifold is considered. This scenario is simulated by introducing an additional airflow rate W_{loss} given in Eq (38). This fault scenario is represented as the change of the value k_{smf} . The fault signal is defined as follows:

$$f(t) = k_{\text{smf}} = \begin{cases} 0, & \text{if } t \in [0, 10] \\ 0.5E-7, & \text{if } t \in [10, 20] \\ 1.5E-7, & \text{else} \end{cases} \quad (40)$$

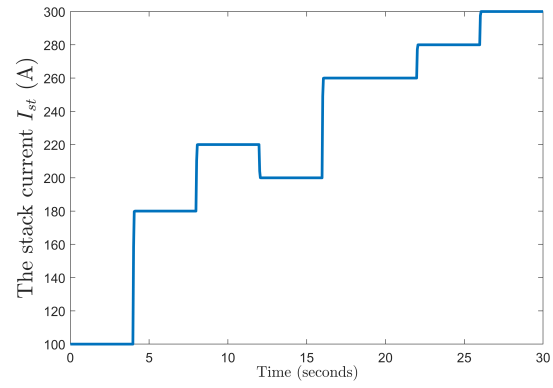


Figure 2: The current I_{st} .

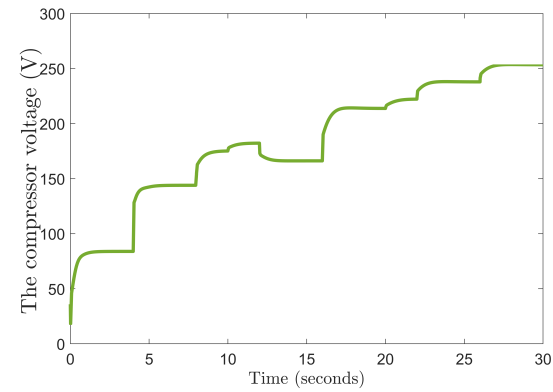


Figure 3: The compressor voltage V_{cm} .

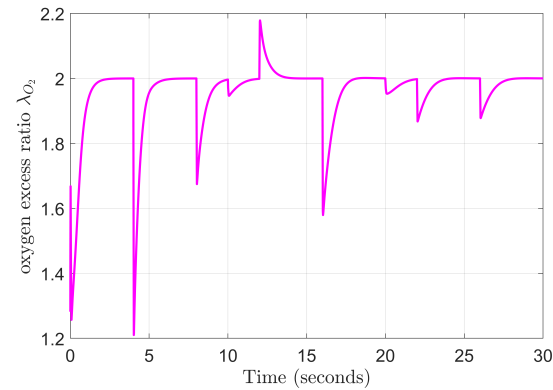


Figure 4: The oxygen excess ratio λ_{O_2} .

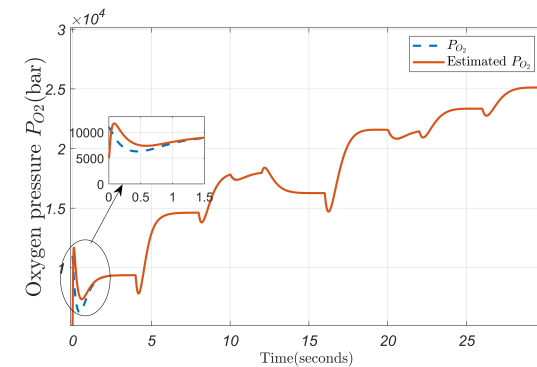


Figure 5: The oxygen pressure p_{O_2} .

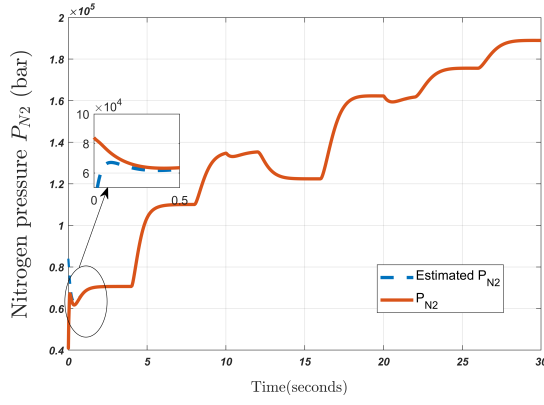
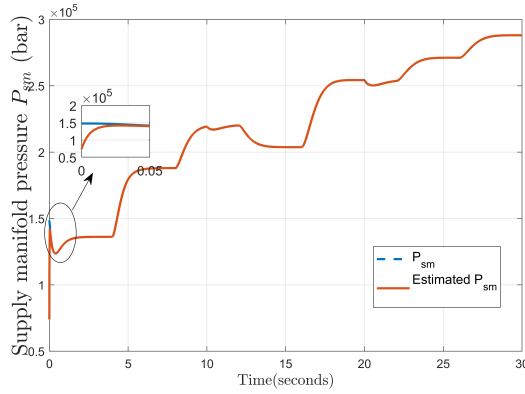
Figure 6: The nitrogen pressure p_{N_2} .

Figure 7: Example of a properly formatted figure

The simulation demonstrated that introducing the leak faults at 10 seconds and 20 seconds had a negligible impact on the performance of the PI controller. This is evident in Figure 3, where the controller's response to the faults can be observed as it compensates for the leakage flow rate. In Figure 4, the rejection of the fault's effect on the oxygen excess ratio is clearly depicted. Furthermore, the observer exhibited remarkable robustness to the fault, as its tracking of the actual values was minimally affected. This robustness underscores the effectiveness of the MHOSMO observer with the selected gains. Figure 8 illustrates the successful reconstruction of the fault. The displayed signal pertains to the W_{loss} value, which allows for the calculation of the leakage percentage relative to the $W_{ca,in}$ flow rate injected into the cathode. Thus, it is evident that the proposed observer is capable of reconstructing the fault signal while simultaneously estimating the system's unmeasured states.

6. Conclusion

This paper proposes a robust fault diagnosis method for the air supply system of PEM fuel cells. Residual signals are generated by comparing the output variables of the fuel cell air supply system model with their corresponding estimates provided by observers. The design of the proposed observer is based on the modified sliding mode super-twisting algorithm, which comprises two nonlinear and two linear terms. This observer is capable of estimating not only the system states but also fault signals in the presence of external dis-

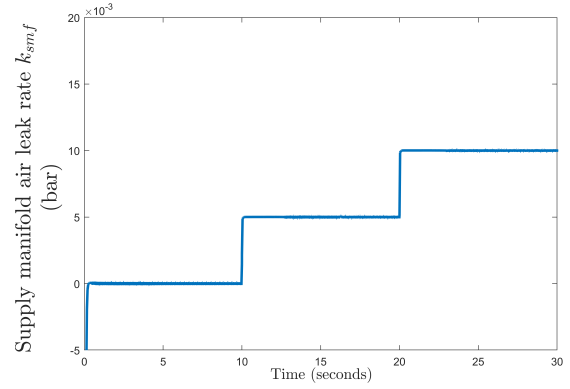
Figure 8: The supply manifold air leak rate k_{smf} .

Table 1: Constants and Parameters Used in the Model

Constant	Expression
c_1	$\frac{RT_{fc}k_{ca,in}}{M_{O_2}V_{ca}} \left(\frac{x_{O_2,atm}}{1 + \omega_{ca,in}} \right)$
c_2	$\frac{p_{sat}}{RT_{fc}}$
c_3	$\frac{V_{ca}}{M_{O_2}}$
c_4	M_{N_2}
c_5	M_{N_2}
c_6	$\frac{M_v p_{sat}}{nRT_{fc}}$
c_7	$\frac{4FV_{ca}}{RT_{fc}k_{ca,in}} \left(\frac{1 - x_{O_2,atm}}{1 + \omega_{atm}} \right)$
c_8	$\frac{f}{M_{N_2}V_{ca}} \left(\frac{1 - x_{O_2,atm}}{1 + \omega_{atm}} \right)$
c_9	$\frac{J_{cp}}{C_p T_{atm}}$
c_{10}	$\frac{J_{cp} \eta_{cp}}{C_p T_{atm}}$
c_{11}	$\frac{p_{atm}}{\gamma - 1}$
c_{12}	$\frac{\gamma}{\eta_{cm} k_t}$
c_{13}	$\frac{J_{cp}}{RT_{atm}}$
c_{14}	$\frac{M_a V_{sm}}{1}$
c_{15}	$\frac{1}{\eta_{cp}}$
c_{16}	$k_{ca,in}$
c_{17}	$\frac{1}{2\pi} \eta_v - c \frac{V_{cpr/tr}}{\rho_a}$
c_{18}	$\frac{k_{ca,in} x_{O_2,atm}}{1 + \omega_{ca,in}}$
c_{19}	$\frac{nM_{O_2}}{4F}$

Table 2: Parameters used in the compressor model.

Symbol	Description	Value
n	Number of cells in the stack	90
γ	Ratio of specific heats of air	1.4
ϕ_{ca}	Relative humidity at the cathode inlet	1.0
ϕ_{atm}	Relative humidity of ambient air	0.5
R	Universal gas constant	8.314 J/(mol.K)
F	Faraday constant	96485 C/mol
p_{atm}	Atmospheric pressure	1.01325×10^5 Pa
T_{fc}	Fuel cell temperature	353.15 K
T_{atm}	Atmospheric temperature	298.15 K
C_p	Specific heat capacity of air	1004 J/(kg.K)
C_D	Nozzle discharge coefficient	0.0038
$x_{O_2,atm}$	Molar ratio of oxygen at the cathode inlet	0.23
M_a	Molar mass of air	28.9644×10^{-3} kg/mol
M_{O_2}	Molar mass of oxygen	32×10^{-3} kg/mol
M_{N_2}	Molar mass of nitrogen	28×10^{-3} kg/mol
M_v	Molar mass of water vapor	18×10^{-3} kg/mol
V_{ca}	Cathode volume	0.01 m ³
V_{sm}	Supply manifold volume	0.02 m ³
A_T	Nozzle opening area	0.002 m ²
J_{cp}	Compressor and motor inertia	67×10^{-9} kg.m ²
η_{cp}	Compressor efficiency	80%
η_{cm}	Mechanical efficiency of the motor	98%
k_t	Motor constant	0.0153 N.m/A
k_v	Motor constant	0.0153 V/(rad/s)
R_{cm}	Motor constant	0.82 Ω
$k_{sm,out}$	Supply manifold outlet constant	0.3629×10^{-5} kg/(Pa.s)
$k_{ca,in}$	Cathode inlet constant	0.3629×10^{-5} kg/(Pa.s)
$k_{ca,out}$	Cathode outlet constant	0.2177×10^{-5} kg/(Pa.s)
ρ_a	Air density	1.23 kg/m ³
$V_{cpr/tr}$	Compressor volume per revolution	5×10^{-4} m ³ /tr
f	Motor friction	0.00136 V/(rad/s)

turbances. Once sliding motion is achieved, the equivalent output error injection term calculated online is used to reconstruct potential system faults. The proposed fault diagnosis approach has been successfully implemented in the Matlab/Simulink environment, where a sudden air leak in the air supply manifold is considered as the fault scenario. Accordingly, the airflow provided by the air compressor must be increased to enhance oxygen flow or the fuel cell operation should be halted immediately if the fault is significant.

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